TREASURY
EXCELLENCE
AS STANDARD

FORMULAE

Advanced Diploma in Treasury Management

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Present value

1. Present value of an annuity: Annuity factor

$$PV = \frac{1-(1+r)^{-n}}{r} \times A_1 \qquad = \frac{A_1}{r}\left(1 - \frac{1}{(1+r)^n}\right) \qquad (= A_1 \text{ x } AF_{(r,n)})$$

$AF_{(r,n)}$ = 1/r x (1-(1+r)$^{-n}$)

= Perpetuity factor x (1 - Discount factor)

Where:
PV = the present (Time 0) value of the annuity
A_1 = the fixed periodic cash flow starting at Time 1 r =
the periodic cost of capital
n = the number of periods to maturity
$AF_{(r,n)}$ = the annuity factor for n periods at a periodic cost of capital of r

2. Present value of a growing perpetuity

$$PV = \frac{A_1}{r - g}$$

Where:
PV = the present (Time 0) value of the perpetuity;
A_1 = the growing periodic cash flow starting at Time 1;
r = the periodic cost of capital; *and*
g = the expected constant future periodic rate of growth of the perpetuity from Time 1.

3. Future value annuity factor (FVAF)

Giving the total future value of an annuity as at Time n periods hence, i.e. the date of the final payment in the series.

FVAF = 1/r x [(1+r)^n - 1]

4. Present value of a growing annuity

$$PV = \frac{A_1}{[r-g]} \times \left(1 - \frac{(1+g)^n}{(1+r)^n}\right)$$

Where:
PV = the present (Time 0) value of the growing annuity;
A_1 = the growing periodic cashflow starting at Time 1;
r = the periodic cost of capital;
g = the constant future periodic rate of growth of the cashflows from Time 1; *and*
n = the total number of cashflows.

5. Dividend growth model for share valuation

$$P_0 = \frac{D_1}{[k_e - g]}$$

Where:
P_0 = the current ex-dividend share price
D_1 = the dividend one period ahead
k_e = the market cost of ordinary share capital
g = the expected constant future dividend growth rate from Time 1 to infinity.

This is a specific application of the general growing perpetuity valuation model (1/[r-g]) above.

Interest rates

6. Continuous compounding

$$FV = PV \times e^{rT}$$

or

$$PV = FV \times e^{-rT}$$

Where:
FV = the future (Time T) value
PV = the present (Time 0) value
e = the exponential constant (2.71828...)
r = continuously compounded rate of return per calendar year
T = number of calendar years

7. Conversion of continuously compounded rate (r) to EAR

$EAR = e^r - 1$

Where:
EAR = effective annual rate of return
e = the exponential constant (2.71828...)
r = continuously compounded rate of return per calendar year

8. Conversion of EAR to a continuously compounded rate (r)

$r = \ln(EAR + 1)$

Where:
EAR = effective annual rate of return
ln = natural logarithm (to the base of e)
r = continuously compounded rate of return per calendar year

Duration

9. Duration

$$D = \frac{\sum_{1=1}^{n} t_i \times PV_i}{\sum_{1=1}^{n} PV_i} = \sum_{1=1}^{n} t_i \times \left(\frac{PV_i}{PV}\right)$$

Where:
D = duration in periods
t_i = period number of flow
PV_i = present value of flow
PV = total present value of all flows in series

Discount factor is either yield to maturity (Macaulay) or zero coupon rates (Fisher-Weil).

10. Modified duration

$$MD = \frac{D}{1 + \frac{YTM}{n}}$$

Where:
MD = modified duration
D = Duration, defined above
YTM = yield to maturity
n = number of coupon periods per year

Options

11. Option pricing: put-call parity relationship

$$S_0 + P - C = Xe^{-rT}$$

For European-style options on a non-income producing underlying asset.

Where:
S_0 = underlying asset market price;
P = Put option value;
C = Call option value;
X = strike price;
r = continuously compounded risk-free rate per calendar year; *and*
T = Time to expiry in calendar years.

Cost of capital

12. Weighted Average Cost of Capital (WACC)

$$WACC = Kd\,x(1-Tc)x\frac{D}{D-C+E} + Kc\,x(1-Tc)x\frac{C}{D-C+E} + Ke\,x\frac{E}{D-C+E}$$

Where:
Kd = market cost of debt
Kc = market return on cash
Ke = cost of equity
TC = corporation rate
D = market value of debt
C = market value of cash
E = market value of equity

13. Dividend growth model for cost of equity

$$Ke = \frac{D_1}{P_0} + g$$

Where:
K_e is the cost of equity
D_1 is the dividend one year hence (usually estimated = D_0 x (1+g))
P_0 is the current ex-dividend share price
g is the expected constant future dividend growth rate from Time 1 to infinity

14. The Capital Asset Pricing Model

$$K_e = R_e = R_f + \beta g\left(R_m - R_f\right)$$

Where:
K_e = cost of equity
R_e = the expected return on a company's share (prospective dividend yield together with expected capital gain)
$_g$ = the share's expected systematic risk
R_m = the return on the market portfolio
R_f = the return on risk-free securities

15. Ungearing and regearing of equity betas (with non-zero debt beta)

$$\text{Assetbeta} = \text{Geared equity beta} \times \frac{MV(E)}{[MV(E) + MV(D)(1-t)]} + \text{Debt beta} \times \frac{MV(D)(1-t)}{[MV(E) + MV(D)(1-t)]}$$

Where:
MV(E) = the market value of equity
MV(D) = the market value of debt
t = the marginal corporate tax rate of relief on debt

When the debt beta is assumed = Nil, then the formula above is simplified as follows:

16. Ungearing and regearing of equity betas (assuming debt beta = Nil)

$$\text{Ungeared beta } \beta u = \text{geared beta } \beta g \times \frac{MV(E)}{[MV(E) + MV(D)(1-t)]}$$

$$\text{Geared beta } \beta_g = \text{ungeared beta } \beta_u \times \frac{[MV(E) + MV(D)(1-t)]}{MV(E)}$$

Where:
Ungeared beta = asset beta
MV(E) = the market value of equity
MV(D) = the market value of debt
t = the marginal corporate tax rate of relief on debt

Statistics and VAR

17. Single assets

Averages: The average value or mean of a population:

$$\mu = \frac{1}{n}\sum_{i=1}^{n}(x_i)$$

Where:
μ = the mean
n = number of observations
x_i = value of each observation

Expected Value: E[X] is the expected value of X. For a frequency distribution describing the whole of the population of possible outturns – for example a normal distribution – the mean is the same as the expected value of an observation.

18. Estimated population variance

$$Var[X] = \frac{1}{n}\sum_{i=1}^{n}(x_i - E[X])^2$$

19. Sample variance

$$Var[X] = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - E[X])^2$$

20. Standard deviation

Var[X] is the same as σ^2.

The standard deviation is the square root of the variance. (A standard normal distribution table is also provided below).

21. The two-asset portfolio

$E[R_p] = w_1 \times \mu_1 + w_2 \times \mu_2$

Where:
$E[R_p]$ = Expected value of return from portfolio
w_1, w_2 = the % weightings of the assets in the portfolio
μ_1, μ_2 = the mean return of each asset in the portfolio

$Var[R_p] = (\sigma[Rp])^2 = w_1^2 \times \sigma_1^2 + w_2^2 \times \sigma_2^2 + 2 \times w_1 \times w_2 \times \sigma_{12}$

Where:
$Var[Rp] = (\sigma[Rp])^2$ = Variance of the return from the portfolio
w_1, w_2 = the % weightings of the assets in the portfolio
σ_1^2, σ_2^2 = the variances of the returns on each portfolio asset
σ_{12} = co-variance between the returns on the portfolio assets

And the standard deviation of the portfolio is the square root of the variance (see also Value at Risk below).

22. Covariance

$$\sigma_{xy} = \frac{\sum\left[(x-\mu_x)(y-\mu_y)\right]}{n}$$

Where:
σ_{xy} = covariance between x and y
μ_x = population x mean
μ_y = population y mean

or

$$\sigma_{xy} = \frac{1}{n}\sum_{i=1}^{n}(r-\mu)\times(r'-\mu')$$

Where:
r = return on asset a at point i
μ = average / expected return on asset a
r' = return on asset b at point i
μ' = average / expected return on asset b
n = number of observations

23. Coefficient of correlation

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

Where:
σ_{xy} = covariance, xy
σ_x = population standard deviation, x
σ_y = population standard deviation, y

24. Value at Risk (VaR)

VaR = nσ * Exposure

Where:
n = number of standard deviations from mean for one-tailed distribution at required confidence level (n approximately =1.65 at 95% confidence, 2.33 at 99% confidence);
σ = standard deviation of % change in value of exposure; *and*
Exposure = value of exposure at current market levels.

25. Correlated Value at Risk

$$VaR_{AB} = \sqrt{VaR_A{}^2 + VaR_B{}^2 + (2 \times \rho_{AB} \times VaR_A \times VaR_B)}$$

Where:
VaR_A is the Value at Risk for variable A
VaR_B is the Value at Risk for variable B
VaR_{AB} is the correlated Value at Risk for variables A and B
$_{AB}$ is the correlation coefficient between variables A and B

26. Financial Valuation

Capex
Replacement capex = Capex charge * (1 + inflation)^k

Where:

$$k = \frac{\text{Accumulated depreciation}}{\text{Depreciation charge}}$$

27. Working Capital

$$\text{WC adjustment} = \frac{\text{WC}}{(1+\text{inflation})} - \text{WC}$$

Where:
WC = working capital level

28. Enterprise Value

$$\text{Enterprise value} = \frac{SCF_1}{\text{WACC} - g}$$

Where:
SCF_1= sustainable cash flow for the following year SCF * (1+g)
g = growth rate in perpetuity

Normal distribution table

Cumulative Distribution Function for the Standard Normal Random Variable [N(x)] where x ≥ 0.

The table shows values of N(x) for x ≥ 0.

The table can be used with interpolation. For example:

$$N(0.4245) = N0.42) + 0.45 \times [N(0.43) - N(0.42)] = 0.663 + 0.45 \times (0.666 - 0.663) = 0.664$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.1	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.2	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.3	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.4	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.5	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.6	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.7	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.8	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811	0.813
0.9	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1.0	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.1	0.864	0.867	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.2	0.885	0.887	0.889	0.891	0.893	0.894	0.896	0.898	0.900	0.901
1.3	0.903	0.905	0.907	0.908	0.910	0.911	0.913	0.915	0.916	0.918
1.4	0.919	0.921	0.922	0.924	0.925	0.926	0.928	0.929	0.931	0.932
1.5	0.933	0.934	0.936	0.937	0.938	0.939	0.941	0.942	0.943	0.944
1.6	0.945	0.946	0.947	0.948	0.949	0.951	0.952	0.953	0.954	0.954
1.7	0.955	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.962	0.963
1.8	0.964	0.965	0.966	0.966	0.967	0.968	0.969	0.969	0.970	0.971
1.9	0.971	0.972	0.973	0.973	0.974	0.974	0.975	0.976	0.976	0.977
2.0	0.977	0.978	0.978	0.979	0.979	0.980	0.980	0.981	0.981	0.982
2.1	0.982	0.983	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986
2.2	0.986	0.986	0.987	0.987	0.987	0.988	0.988	0.988	0.989	0.989
2.3	0.989	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.992
2.4	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994
2.5	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995	0.995	0.995
2.6	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996
2.7	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
2.8	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
2.9	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3.0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000